Examples of Dynamical Systems and their corresponding Limit Spaces, End Spaces and Decompositions

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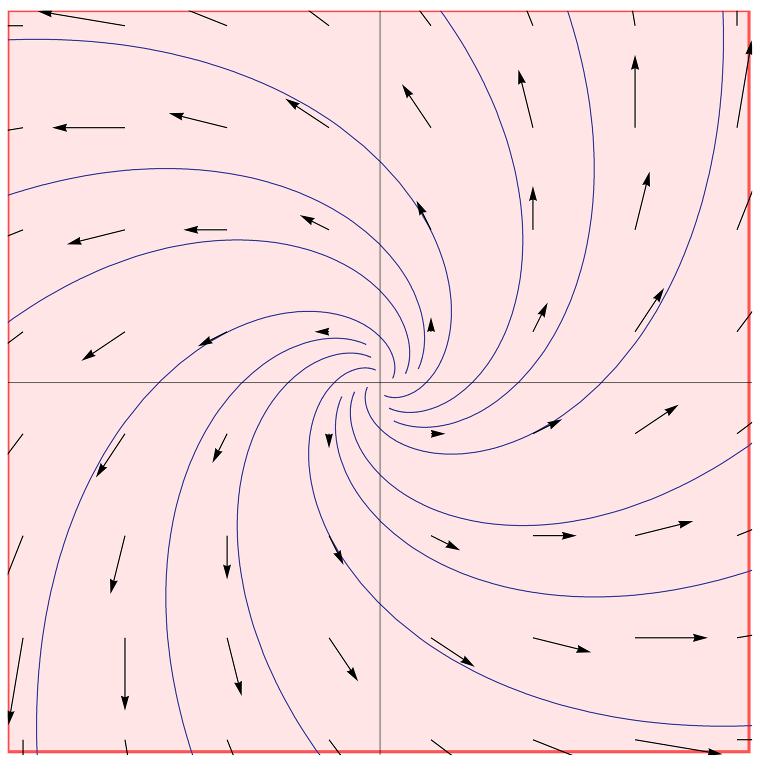
1. Dynamical systems on $\mathbb{C}$ and $S^2 = \mathbb{C} \cup \{\infty\}$:

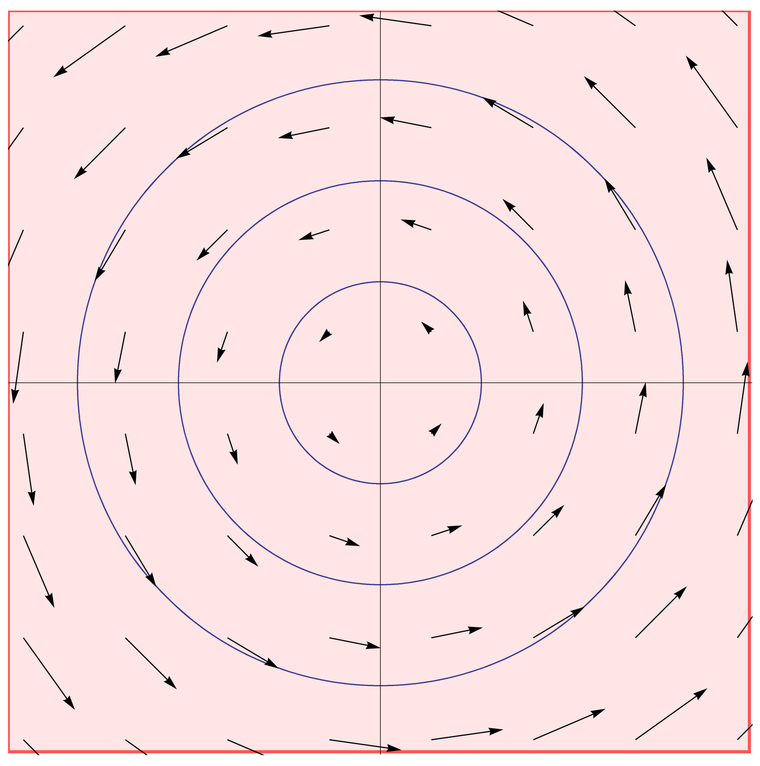
1.1. $\phi\colon \mathbb{R} \times \mathbb{C} \to \mathbb{C}, \phi(r, u) = e^{rz}u$, $\phi(r, \infty) = \infty$ $(z \in \mathbb{C},\ Re(z) \neq 0)$



$(\mathbb{C}, \phi)$		
	$L^{\mathbf{r}} = \{0\}$	$\check{\pi}_0^{\mathbf{r}} = \{0, \infty\}$
	$L_0^{\mathbf{r}} = \{0\}$	$L_\infty^{\mathbf{r}} = \emptyset$
	$X_{(\mathbf{r},0)} = \{0\}$	$X_{(\mathbf{r},\infty)} = \mathbb{C} \setminus \{0\}$
	$L^{\mathbf{l}} = \{0\}$	$\check{\pi}_0^{\mathbf{l}} = \{0\}$
	$L_0^{\mathbf{l}} = \{0\}$	
	$X_{(\mathbf{l},0)} = \mathbb{C}$	
(S^2, ϕ)		
	$L^{\mathbf{r}} = \{0, \infty\}$	$\check{\pi}_0^{\mathbf{r}} = \{0, \infty\}$
	$L_0^{\mathbf{r}} = \{0\}$	$L_\infty^{\mathbf{r}} = \{\infty\}$
	$X_{(\mathbf{r},0)} = \{0\}$	$X_{(\mathbf{r},\infty)} = S^2 \setminus \{0\}$
	$L^{\mathbf{l}} = \{0, \infty\}$	$\check{\pi}_0^{\mathbf{l}} = \{0, \infty\}$
	$L_0^{\mathbf{l}} = \{0\}$	$L_\infty^{\mathbf{l}} = \{\infty\}$
	$X_{(\mathbf{l},0)} = S^2 \setminus \{\infty\}$	$X_{(\mathbf{l},\infty)} = \{\infty\}$

1.2. $\varphi\colon \mathbb{R} \times \mathbb{C} \to \mathbb{C}, \varphi(r, u) = e^{rz}u$, $\varphi(r, \infty) = \infty$ $(0 \neq z \in \mathbb{C},\ Re(z) = 0)$

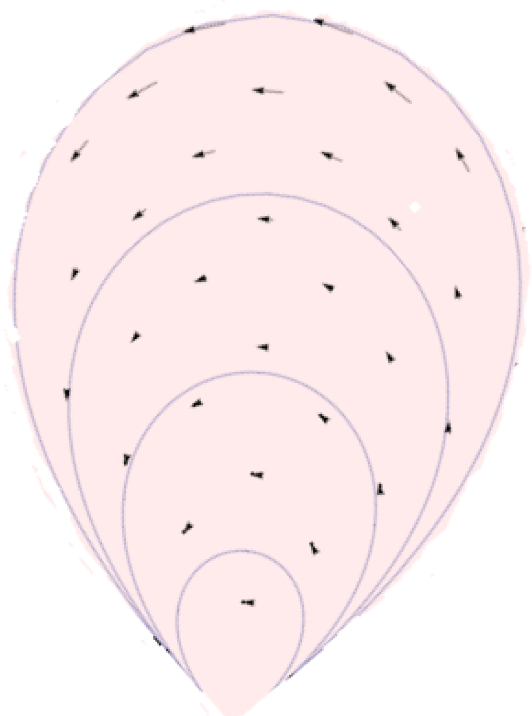
$(\mathbb{C}, \varphi)$		
	$L^{\mathbf{r}} = \mathbb{C}$	$\check{\pi}_0^{\mathbf{r}} = \{*\}$
	$L_*^{\mathbf{r}} = \mathbb{C}$	
	$X_{(\mathbf{r},*)} = \mathbb{C}$	
	$L^{\mathbf{l}} = \mathbb{C}$	$\check{\pi}_0^{\mathbf{l}} = \{*\}$
	$L_*^{\mathbf{l}} = \mathbb{C}$	
	$X_{(\mathbf{l},*)} = \mathbb{C}$	
(S^2, φ)		
	$L^{\mathbf{r}} = S^2$	$\check{\pi}_0^{\mathbf{r}} = \{*\}$
	$L_*^{\mathbf{r}} = S^2$	
	$X_{(\mathbf{r},*)} = S^2$	
	$L^{\mathbf{l}} = S^2$	$\check{\pi}_0^{\mathbf{l}} = \{*\}$
	$L_*^{\mathbf{l}} = S^2$	
	$X_{(\mathbf{l},*)} = S^2$	



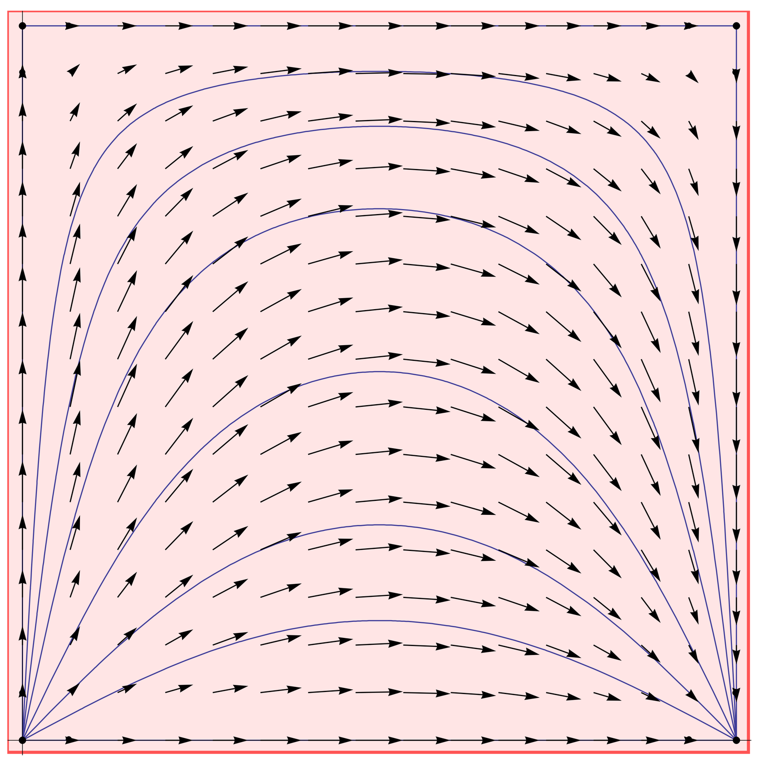
2. Dynamical systems on 2-cells:

2.1. $\phi\colon \mathbb{R} \times D^2 \to D^2$ **with one critical point** P

$L^{\mathbf{r}} = \{P\}$	$\check{\pi}_0^{\mathbf{r}} = \{P\}$
$L_P^{\mathbf{r}} = \{P\}$	
$X_{(\mathbf{r},P)} = D^2$	
$L^{\mathbf{l}} = \{P\}$	$\check{\pi}_0^{\mathbf{l}} = \{P\}$
$L_P^{\mathbf{l}} = \{P\}$	
$X_{(\mathbf{l},P)} = D^2$	



2.2.



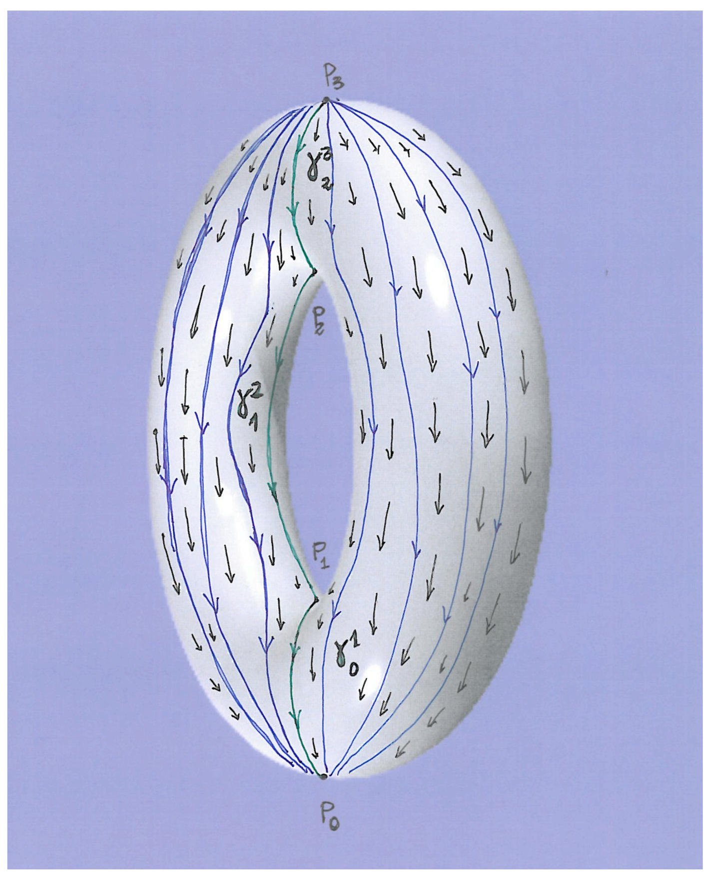
$\varphi\colon \mathbb{R} \times ([0,1] \times [0,1]) \to [0,1] \times [0,1]$ **with four critical points** $P_0^0 = (0,0), P_0^1 = (0,1), P_1^1 = (1,1), P_1^0 = (1,0)$

$L^{\mathbf{r}} = \{P_0^0, P_0^1, P_1^1, P_1^0\}$	$\check{\pi}_0^{\mathbf{r}} = \{P_0^0, P_0^1, P_1^1, P_1^0\}$
$L_{P_i^j}^{\mathbf{r}} = \{P_i^j\}$	
$X_{(\mathbf{r},P_0^0)} = \{P_0^0\}$	$X_{(\mathbf{r},P_0^1)} = \{0\} \times (0,1]$
$X_{(\mathbf{r},P_1^1)} = (0,1] \times \{1\}$	$X_{(\mathbf{r},P_1^0)} = (0,1] \times [0,1)$
$L^{\mathbf{l}} = \{P_0^0, P_0^1, P_1^1, P_1^0\}$	$\check{\pi}_0^{\mathbf{l}} = \{P_0^0, P_0^1, P_1^1, P_1^0\}$
$L_{P_i^j}^{\mathbf{l}} = \{P_i^j\}$	
$X_{(\mathbf{l},P_1^0)} = \{P_1^0\}$	$X_{(\mathbf{l},P_1^1)} = \{1\} \times (0,1]$
$X_{(\mathbf{l},P_0^1)} = [0,1) \times \{1\}$	$X_{(\mathbf{l},P_0^0)} = [0,1) \times [0,1)$

3. Dynamical systems on a torus and a cylinder:

3.1. $\phi\colon \mathbb{R} \times (S^1 \times S^1) \to S^1 \times S^1$ **with four critical points (induced by the gradient of the height function)**

$L^{\mathbf{r}} = \{P_0, P_1, P_2, P_3\}$	$\check{\pi}_0^{\mathbf{r}} = \{P_0, P_1, P_2, P_3\}$
$L_{P_i}^{\mathbf{r}} = \{P_i\}$	
$X_{(\mathbf{r},P_3)} = \{P_3\}$	$X_{(\mathbf{r},P_2)} = \{P_2\} \cup \gamma_2^3 \cup \tilde{\gamma}_2^3$
$X_{(\mathbf{r},P_1)} = \{P_1\} \cup \gamma_1^2 \cup \tilde{\gamma}_1^2$	
$X_{(\mathbf{r},P_0)} = (S^1 \times S^1) \setminus \bigcup_{i=1}^3 X_{(\mathbf{r},P_i)}$	
$L^{\mathbf{l}} = \{P_0, P_1, P_2, P_3\}$	$\check{\pi}_0^{\mathbf{l}} = \{P_0, P_1, P_2, P_3\}$
$L_{P_i}^{\mathbf{l}} = \{P_i\}$	
$X_{(\mathbf{l},P_0)} = \{P_0\}$	$X_{(\mathbf{l},P_1)} = \{P_1\} \cup \gamma_0^1 \cup \tilde{\gamma}_0^1$
$X_{(\mathbf{l},P_2)} = \{P_2\} \cup \gamma_1^2 \cup \tilde{\gamma}_1^2$	
$X_{(\mathbf{l},P_3)} = (S^1 \times S^1) \setminus\ \bigcup_{i=0}^2 X_{(\mathbf{l},P_i)}$	



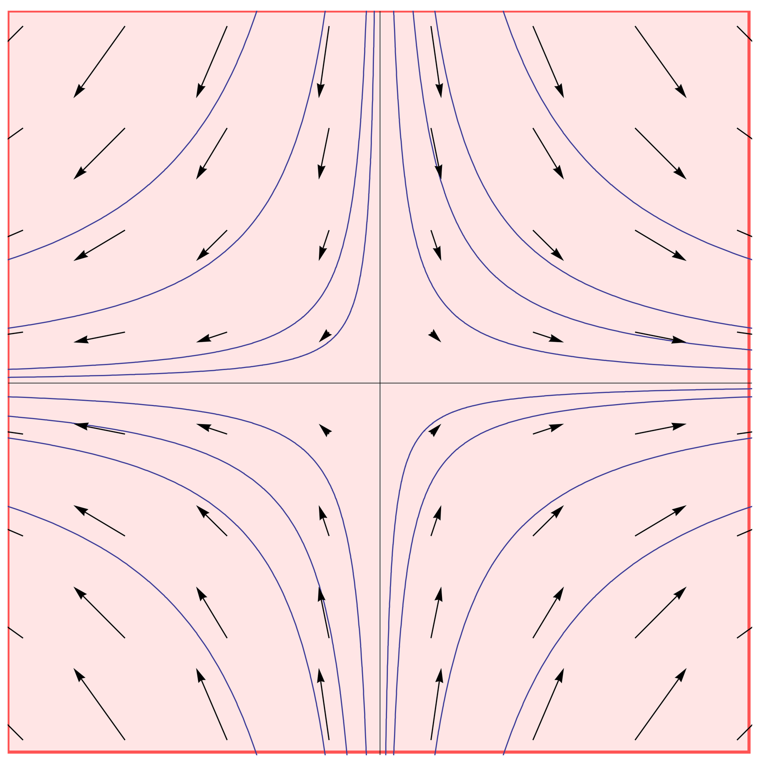
3.2.

$\varphi\colon \mathbb{R} \times (S^1 \times \mathbb{R}) \to S^1 \times \mathbb{R}$ **with two periodic trajectories**

$L^{\mathbf{r}} = \gamma^{-1} \cup \gamma^1$	$\check{\pi}_0^{\mathbf{r}} = \{-\infty, *_{-1}, *_1, +\infty\}$
$L_{-\infty}^{\mathbf{r}} = \emptyset$	$L_{*_{-1}}^{\mathbf{r}} = \gamma^{-1}$
$L_{*_1}^{\mathbf{r}} = \gamma^1$	$L_{+\infty}^{\mathbf{r}} = \emptyset$
$X_{(\mathbf{r},-\infty)} = S^1 \times (-\infty, -1)$	$X_{(\mathbf{r},*_{-1})} = S^1 \times [-1, 1)$
$X_{(\mathbf{r},*_1)} = S^1 \times \{1\}$	$X_{(\mathbf{r},-\infty)} = S^1 \times (1, +\infty)$
$L^{\mathbf{l}} = \gamma^{-1} \cup \gamma^1$	$\check{\pi}_0^{\mathbf{l}} = \{*_{-1}, *_1\}$
$L_{*_{-1}}^{\mathbf{l}} = \gamma^{-1}$	$L_{*_1}^{\mathbf{l}} = \gamma^1$
$X_{(\mathbf{l},*_{-1})} = S^1 \times (-\infty, -1]$	$X_{(\mathbf{l},*_1)} = S^1 \times (-1, +\infty)$

4. Dynamical systems on $\mathbb{R}^2$ and $S^2 = \mathbb{R}^2 \cup \{\infty\}$:

4.1. $\phi\colon \mathbb{R} \times \mathbb{R}^2 \to \mathbb{R}^2$, $\phi(r, (u_1, u_2)) = (e^{r\lambda_1}u_1, e^{r\lambda_2}u_2)$, $\phi(r, \infty) = \infty$ $(\lambda_1 > 0,\ \lambda_2 < 0)$



$(\mathbb{R}^2, \phi)$		
	$L^{\mathbf{r}} = \{0\}$	$\check{\pi}_0^{\mathbf{r}} = \{\infty_-, 0, \infty_+\}$
	$L_0^{\mathbf{r}} = \{0\}$	$L_{\infty_-}^{\mathbf{r}} = L_{\infty_+}^{\mathbf{r}} = \emptyset$
	$X_{(\mathbf{r},0)} = \{0\} \times \mathbb{R}$	$X_{(\mathbf{r},\infty_+)} = \mathbb{R}_+ \times \mathbb{R}$, $X_{(\mathbf{r},\infty_-)} = \mathbb{R}_- \times \mathbb{R}$
	$L^{\mathbf{l}} = \{0\}$	$\check{\pi}_0^{\mathbf{l}} = \{\infty^-, 0, \infty^+\}$
	$L_0^{\mathbf{l}} = \{0\}$	$L_{\infty^-}^{\mathbf{l}} = L_{\infty^+}^{\mathbf{l}} = \emptyset$
	$X_{(\mathbf{l},0)} = \mathbb{R} \times \{0\}$	$X_{(\mathbf{l},\infty^+)} = \mathbb{R} \times \mathbb{R}_+$, $X_{(\mathbf{l},\infty^-)} = \mathbb{R} \times \mathbb{R}_-$
(S^2, ϕ)		
	$L^{\mathbf{r}} = \{0, \infty\}$	$\check{\pi}_0^{\mathbf{r}} = \{0, \infty\}$
	$L_0^{\mathbf{r}} = \{0\}$	$L_\infty^{\mathbf{r}} = \{\infty\}$
	$X_{(\mathbf{r},0)} = \{0\} \times \mathbb{R}$	$X_{(\mathbf{r},\infty)} = ((\mathbb{R} \setminus \{0\}) \times \mathbb{R}) \cup \{\infty\}$
	$L^{\mathbf{l}} = \{0, \infty\}$	$\check{\pi}_0^{\mathbf{l}} = \{0, \infty\}$
	$L_0^{\mathbf{l}} = \{0\}$	$L_\infty^{\mathbf{l}} = \{\infty\}$
	$X_{(\mathbf{l},0)} = \mathbb{R} \times \{0\}$	$X_{(\mathbf{l},\infty)} = (\mathbb{R} \times (\mathbb{R} \setminus \{0\})) \cup \{\infty\}$

$(\mathbb{R}_+ = \{r \in \mathbb{R} | r > 0\}, \mathbb{R}_- = \{r \in \mathbb{R} | r < 0\})$

4.2. $\varphi\colon \mathbb{R} \times \mathbb{R}^2 \to \mathbb{R}^2$, $\varphi(r, (u_1, u_2)) = (r + u_1, u_2)$, $\varphi(r, \infty) = \infty$

$(\mathbb{R}^2, \varphi)$		
	$L^{\mathbf{r}} = \emptyset$	$\check{\pi}_0^{\mathbf{r}} = \{+\infty\}$
	$L_{+\infty}^{\mathbf{r}} = \emptyset$	
	$X_{(\mathbf{r},+\infty)} = \mathbb{R}^2$	
	$L^{\mathbf{l}} = \emptyset$	$\check{\pi}_0^{\mathbf{l}} = \{-\infty\}$
	$L_{-\infty}^{\mathbf{l}} = \emptyset$	
	$X_{(\mathbf{l},-\infty)} = \mathbb{R}^2$	
(S^2, φ)		
	$L^{\mathbf{r}} = \{\infty\}$	$\check{\pi}_0^{\mathbf{r}} = \{\infty\}$
	$L_\infty^{\mathbf{r}} = \{\infty\}$	
	$X_{(\mathbf{r},\infty)} = S^2$	
	$L^{\mathbf{l}} = \{\infty\}$	$\check{\pi}_0^{\mathbf{l}} = \{\infty\}$
	$L_\infty^{\mathbf{l}} = \{\infty\}$	
	$X_{(\mathbf{l},\infty)} = S^2$