

An introduction to Homotopy QFTs, talks by Tim Porter.

Summary:

1. Brief ‘recall’ of simplicial set theory (simplicial sets, simplicial groups, some examples from geometric topology, Moore complexes, homotopy groups etc., bundles and classifying spaces, \overline{W} , ‘homotopy finite’);
2. Crossed modules and crossed complexes; their relation with simplicial groups (2-types and generally n -types). Crossed modules and categorical groups (a special type of strict monoidal category). Nerves for crossed modules, etc. and their relationship with \overline{W} ;
3. Yetter’s construction of TQFTs from a finite crossed module, extension to n -types (1997);
4. Relative theory (1998): $G \rightarrow H$ morphism of simplicial groups with sufficiently (homotopy) finite kernel, possibly interesting examples: essentially derived from $PL \rightarrow Top$, $O \rightarrow PL$, $Spin \rightarrow SO$ (other interesting cases???)
5. HQFT (Turaev 1998) then Rodrigues (2000), idea *an HQFT is a TQFT with ‘background’*, Theorem of Rodrigues : HQFT based on d -dimensional manifolds with background B , depends only on the $(d + 1)$ -type of B :
question: *take an algebraic model for an $(d + 1)$ -type, can we construct a HQFT with background its classifying space? And can we classify these HQFTs?*
6. Classification : Case 1 (Turaev) $B = K(\pi, 1)$, $d = 1$ solution crossed π -algebra;
Case 2. (Brightwell-Turner, 2000) $B = K(A, 2)$, $d = 1$ solution Frobenius algebra with action of A ;
7. $1 + 1$ -dim FHQFT (so again $d = 1$) (Porter-Turaev, 2003-2005) Crossed \mathcal{C} -algebras,
8. Links with relative TQFT situations?
9. HQFTs classify what?

Plan: sections 1-4 in first session, the remainder in the second.