SCATTERING MATRIX IN *DKP* THEORY: BARRIER POTENTIAL CASE

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INTRODUCTION

The *DKP* equation: $$[i\beta^{\mu}(\partial_{\mu} + ieA_{\mu}) - m]\psi(\mathbf{r},t) = 0$$

$$\mathbf{r} = (x,y,z) \qquad \partial_{\mu} = (\frac{\partial}{\partial t},\boldsymbol{\nabla}) \qquad A^{\mu} = (V,A)$$

The β^{μ}verify the *DKP* algebra: $$\beta^{\mu}\beta^{\nu}\beta^{\lambda} + \beta^{\lambda}\beta^{\nu}\beta^{\mu} = g^{\mu\nu}\beta^{\lambda} + g^{\nu\lambda}\beta^{\mu}$$

$$g^{\mu\nu} = diag(1,-1,-1,-1)$$

The continuity equation: $$\partial_{\mu}J^{\mu} = 0$$

$$J^{\mu} = (J^{0},J^{k}) = \overline{\psi}\beta^{\mu}\psi$$

The charge density : $$J^{0} = \overline{\psi}\beta^{0}\psi$$

The current density: $$J^{k} = \overline{\psi}\beta^{k}\psi$$

The adjoint: $$\overline{\psi} = \psi^{+}\left[2(\beta^{0})^{2} - 1\right]$$

For the scalar particles:

$$\beta^0 = \begin{pmatrix} \boldsymbol{\theta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}; \quad \beta^i = \begin{pmatrix} \mathbf{0} & \rho^i \\ -\rho_T^i & \mathbf{0} \end{pmatrix} ; i = 1,2,3$$

$$\rho^1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \rho^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \rho^3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \boldsymbol{\theta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

For the vectorial particles:

$$\beta^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \bar{0}^T & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}; \quad \beta^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{0} & -is_i \\ -e_i^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{0}^T & -is_i & \mathbf{0} & \mathbf{0} \end{pmatrix} ; i = 1,2,3$$

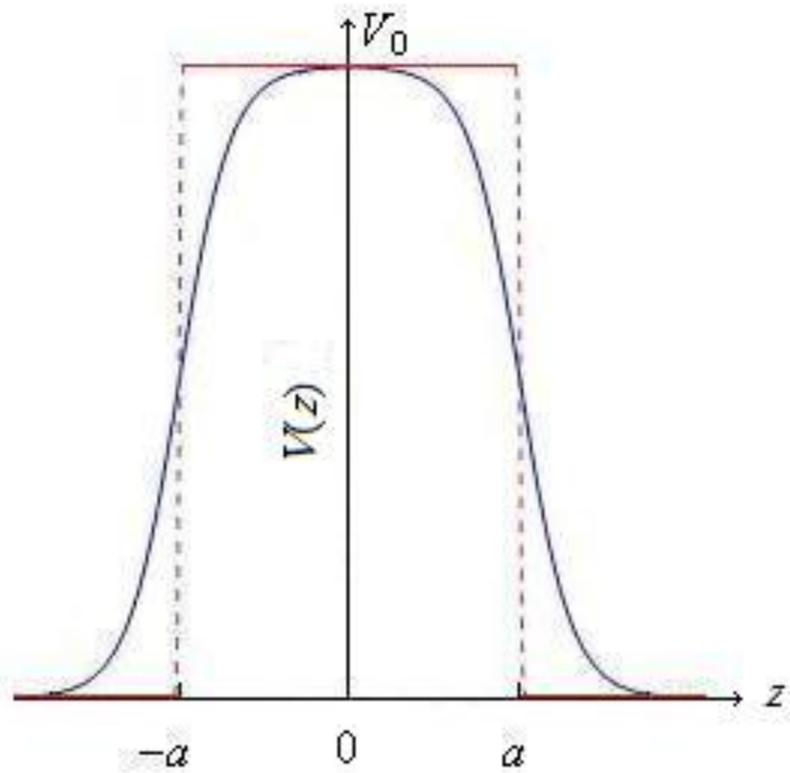
$$e_1 = (1,0,0); e_2 = (0,1,0); e_3 = (0,0,1); \bar{0} = (0,0,0)$$

The scalar potential of WS:

$$V(z) = \frac{V_0}{1+\exp\left(\frac{|z|-a}{r}\right)}$$

The barrier potential:

$$\lim_{r\to 0^+} V(z) = V_0\theta(a-|z|)$$



RESOLUTION OF THE DKP EQUATION IN THE SCALAR POTENTIEL OF WS, SPIN 1

DKP equation: $$\left[\beta^0(E-eV)+i\beta^3\frac{d}{dz}-m\right]\widetilde{\phi(z)}=0$$

$$\widetilde{\phi(z)}^T=(\varphi,\mathbf{A},\mathbf{B},\mathbf{C})$$

Knowing that: $$\begin{cases}\mathbf{O}_{KG}\Psi=0\\ \Phi=\frac{E-eV}{m}\Psi\\ \Theta=\frac{i}{m}\frac{d}{dz}\Psi\end{cases}$$

where: $$\begin{cases}\Psi^T=(A_1,A_2,B_3)\\ \Phi^T=(B_1,B_2,A_3)\\ \Theta^T=(C_2,-C_1,\varphi)\end{cases}$$

$$O_{KG}=\frac{d^2}{dz^2}+\left[(E-eV)^2-m^2\right]$$

we designate by $\phi(z)$ the solution of the *DKP* equation :

$$\phi(z)^T=(\Psi,\Phi,\Theta)$$

The asymptotic forms of the wave function :

$$\lim_{z\to-\infty} \phi_L(z) = Ae^{-ik(z+a)}\begin{pmatrix} 1 \\ \frac{E}{m} \\ \frac{k}{m} \end{pmatrix} \otimes \mathbf{V} + Be^{ik(z+a)}\begin{pmatrix} 1 \\ \frac{E}{m} \\ -\frac{k}{m} \end{pmatrix} \otimes \mathbf{V}$$

$$\lim_{z\to+\infty} \phi_R(z) = Ce^{ik(z-a)}\begin{pmatrix} 1 \\ \frac{E}{m} \\ -\frac{k}{m} \end{pmatrix} \otimes \mathbf{V} + De^{-ik(z-a)}\begin{pmatrix} 1 \\ \frac{E}{m} \\ \frac{k}{m} \end{pmatrix} \otimes \mathbf{V}$$

$\mathbf{V}$:constant vector related to the 3 directions of spin1: $\mathbf{V} = \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}$

The charge density: $J = \overline{\psi}\beta^3\psi$

Along each direction of the spin V_i, $i = 1,2,3$

$$\begin{cases} J_{iL} = J_{inc} - J_{ref} = 2N_i^2 \frac{k}{m}\left[|B|^2 - |A|^2\right] \\ J_{iR} = J_{trans} = 2N_i^2 \frac{k}{m}|C|^2 \end{cases}$$

The reflection and transmission coefficients:

$$\mathbf{R} = \frac{|J_{ref}|}{|J_{inc}|} = \frac{|A|^2}{|B|^2}, \mathbf{T} = \frac{|J_{trans}|}{|J_{inc}|} = \frac{|C|^2}{|B|^2}$$

In the barrier case:

$$\mathbf{R} = \frac{|J_{ref}|}{|J_{inc}|} = \frac{\left(\frac{k^2 - p^2}{2pk}\right)^2 \sin^2 2pa}{1+\left(\frac{k^2 - p^2}{2pk}\right)^2 \sin^2 2pa}$$

$$\mathbf{T} = \frac{|J_{trans}|}{|J_{inc}|} = \frac{1}{1+\left(\frac{k^2 - p^2}{2pk}\right)^2 \sin^2 2pa}$$

$$p^2 = (E - eV_0)^2 - m^2, k^2 = E^2 - m^2$$

RESOLUTION OF THE DKP EQUATION IN THE SCALAR POTENTIEL OF WS, SPIN 0

The Wave function: $\widetilde{\phi(z)}^T = (\eta_1, \eta_2, \eta_3, \eta_4, \eta_5)$ With: $\eta_3 = \eta_4 = 0$

The solution of the *DKP* equation: $\phi(z)^T = (\eta_1, \eta_2, \eta_5)$

Such as:

Spin1: $\mathbf{O}_{KG}\Psi = 0; \Phi = \frac{E-eV}{m}\Psi; \Theta = \frac{i}{m}\frac{d}{dz}\Psi$

Spin0: $\mathbf{O}_{KG}\eta_1 = 0; \eta_2 = \frac{E-eV}{m}\eta_1; \eta_5 = \frac{i}{m}\frac{d}{dz}\eta_1$

We make these correspondances: $\Psi \to \eta_1; \eta_2 \to \Phi; \eta_5 \to \Theta$

and we get the same coefficients R and T as for the spin 1.

SCATTERING MATRIX, SPIN 1

The incoming and the outgoing parts of the wave function:

$$\begin{cases} \phi_{\pm}^{in}(z) = \theta(-z)e^{\pm ik(z+a)}M \otimes \mathbf{V} + \theta(z)e^{\mp ik(z-a)}N \otimes \mathbf{V} \\ \phi_{\pm}^{out}(z) = \theta(z)e^{\pm ik(z-a)}M' \otimes \mathbf{V} + \theta(-z)e^{\mp ik(z+a)}N' \otimes \mathbf{V} \end{cases}$$

Where:

$$\phi_{-}(z) : -\infty \to +\infty$$

$$\phi_{+}(z) : +\infty \to -\infty$$

$$M = \begin{pmatrix} B \\ B\frac{E}{m} \\ -B\frac{k}{m} \end{pmatrix}; M' = \begin{pmatrix} C \\ C\frac{E}{m} \\ -C\frac{k}{m} \end{pmatrix}; N = \begin{pmatrix} D \\ D\frac{E}{m} \\ D\frac{k}{m} \end{pmatrix}; N' = \begin{pmatrix} A \\ A\frac{E}{m} \\ A\frac{k}{m} \end{pmatrix}$$

The S matrix: $$S = \begin{pmatrix} T & R \\ R & T \end{pmatrix} \otimes I_9$$

$$\begin{pmatrix} M' \\ N' \end{pmatrix} \otimes \mathbf{V} = S\left[\begin{pmatrix} M \\ N \end{pmatrix} \otimes \mathbf{V}\right]$$

The pseudo-unitarity of *S*: $$S\bar{S} = \bar{S}S = \overline{I_{18}}$$

The pseudo-adjoint matrix: $$\bar{S} = \left(I_2 \otimes \widetilde{\beta^0}\right) S^{+} \left(I_2 \otimes \widetilde{\beta^0}\right)$$

$$\widetilde{\beta^0} = \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}; \overline{I_{18}} = \left(I_2 \otimes \widetilde{\beta^0}\right)^2$$

SCATTERING MATRIX, SPIN 0

The incoming and the outgoing parts of the wave function:

$$\begin{cases} \phi_{\pm}^{in}(z) = \theta(-z)e^{\pm ik(z+a)}M + \theta(z)e^{\mp ik(z-a)}N \\ \phi_{\pm}^{out}(z) = \theta(z)e^{\pm ik(z-a)}M' + \theta(-z)e^{\mp ik(z+a)}N' \end{cases}$$

The S matrix: $S = \begin{pmatrix} T & R \\ R & T \end{pmatrix} \otimes I_3 \qquad \begin{pmatrix} M' \\ N' \end{pmatrix} = S\begin{pmatrix} M \\ N \end{pmatrix}$

The pseudo-unitarity of S: $S\bar{S} = \bar{S}S = \overline{I_6}$

The pseudo-adjoint matrix: $\bar{S} = \left(I_2 \otimes \widetilde{\beta^0}\right)S^+\left(I_2 \otimes \widetilde{\beta^0}\right)$

$$\widetilde{\beta^0} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \overline{I_6} = \left(I_2 \otimes \widetilde{\beta^0}\right)^2$$

CONCLUSION

We have constructed the *DKP* scattering matrix for time-like Lorentz vector interaction. We haven't discussed in this context the boundary conditions which are intimately related to the Klein tunneling. It would be very interesting to establish a relation between them via the Green function. This question has been treated in a submitted publication.